Series - Semiconductor physics and light matter interaction

Series 7 - Single level recombination and generation.

1. a) The occupation of the defect levels will be controlled by the Fermi-Dirac distribution. N.B - We CANNOT use the Boltzmann approximation since we know the defect level energy can be near the Fermi energy, where the Boltzmann approximation breaks down.

$$f_t = \frac{1}{1+e^{(E_t-E_F)/k_BT}}$$, occupation probability at $E = E_t$, the defect level energy.

$$\Rightarrow \boxed{N_t^- = N_t f_t} \quad (1) \qquad \boxed{N_t^x = N_t(1-f_t)} \quad (2)$$

b) Thermal equilibrium - in the steady state, generation = recombination
∴ emission = capture for both electrons and holes:

$$r_{c,n} = r_{e,n} \quad , \quad r_{c,p} = r_{e,p}$$ (Ignoring band to band & Auger)

$$\therefore n\sigma_n v_{th} N_t^x = e_n N_t^- \quad , \quad p\sigma_p v_{th} N_t^- = e_p N_t^x$$

Using (1) and (2):

$$n\sigma_n v_{th} \cancel{N_t}(1-f_t) = e_n \cancel{N_t} f_t$$
$$p\sigma_p v_{th} \cancel{N_t} f_t = e_p \cancel{N_t}(1-f_t)$$

$$\therefore e_n = \sigma_n v_{th} n\left(\frac{1-f_t}{f_t}\right) \quad , \quad e_p = \sigma_p v_{th} p\left(\frac{f_t}{1-f_t}\right)$$

$$\frac{f_t}{1-f_t} = \left(\frac{1}{\cancel{1+e^{(E_t-E_F)/k_BT}}}\right)\left(\frac{\cancel{1+e^{(E_t-E_F)/k_BT}}}{e^{(E_t-E_F)/k_BT}}\right) = e^{-(E_t-E_F)/k_BT}$$

$$\therefore e_n = \sigma_n v_{th} n e^{(E_t-E_F)/k_BT} \quad (3) \quad , \quad e_p = \sigma_p v_{th} p e^{(E_F-E_t)/k_BT} \quad (4)$$

Now, we can use the Boltzmann approximation to obtain expressions for n and p, this just means we are assuming the Fermi level is far from the conduction band / valence band edge - i.e. we are thinking of a non-degenerate, LIGHTLY doped semiconductor.

$$\therefore n = N_C e^{-(E_C-E_F)/k_BT} \quad , \quad p = N_V e^{-(E_F-E_V)/k_BT}$$

Also, as always, we can express the intrinsic carrier density also using the Boltzmann approximation:

$$n_i = N_C e^{-(E_C-E_{F,i})/k_BT} = N_V e^{-(E_{F,i}-E_V)/k_BT}$$

Combining these two expressions:

$n = n_i e^{(E_F - E_{F,i})/k_BT}$, $p = n_i e^{(E_{F,i} - E_F)/k_BT}$, sub into ③ & ④ :

$\therefore\ e_n = \sigma_n v_{th} n_i e^{(E_F - E_{F,i})/k_BT} e^{(E_t - E_F)/k_BT}$, $e_p = \sigma_p v_{th} n_i e^{(E_{F,i} - E_F)/k_BT} e^{(E_F - E_t)/k_BT}$

Hence $e_n = \sigma_n v_{th} n_i e^{(E_t - E_{F,i})/k_BT}$, $e_p = \sigma_p v_{th} n_i e^{(E_{F,i} - E_t)/k_BT}$

$\therefore\ \boxed{e_n = \sigma_n v_{th} n_t}$ ⑤ , $\boxed{e_p = \sigma_p v_{th} p_t}$ ⑥

So, as we could expect, emission probability exponentially increases as the defect level energy drifts further away from the intrinsic Fermi Level (i.e., closer to the conduction band / valence band).

SUPPLEMENTARY INFORMATION: DERIVATION OF EQ. (1)

Considering the equilibrium steady state, $\frac{dn}{dt} = \frac{dp}{dt} = 0$

As such, $G_L = R_n = R_p = R = \underbrace{r_{c,n} - r_{e,n} = r_{c,p} - r_{e,p}}_{⑦}$

Using ①, ②, ⑤, ⑥ we can get full expressions for $r_{c,n}$, $r_{e,n}$, $r_{c,p}$, $r_{e,p}$:

$r_{c,n} = \sigma_n v_{th} n N_t (1 - f_t)$, $r_{e,n} = \sigma_n v_{th} n_t N_t f_t$

$r_{c,p} = \sigma_p v_{th} p N_t f_t$, $r_{e,p} = \sigma_p v_{th} p_t N_t (1 - f_t)$

Subbing into ⑦, we can get an expression for f_t:

$$\sigma_n \cancel{v_{th} N_t}[n(1 - f_t) - n_t f_t] = \sigma_p \cancel{v_{th} N_t}[p f_t - p_t(1 - f_t)]$$

$$\therefore\ \sigma_n n - \sigma_n n f_t - \sigma_n n_t f_t = \sigma_p p f_t - \sigma_p p_t + \sigma_p p_t f_t$$

$$\therefore\ f_t[\sigma_n(n + n_t) + \sigma_p(p + p_t)] = \sigma_n n + \sigma_p p_t$$

$$\therefore\ f_t = \frac{\sigma_n n + \sigma_p p_t}{\sigma_n(n + n_t) + \sigma_p(p + p_t)}$$

→ We can now sub this into one side of ⑦, e.g. $R = r_{c,n} - r_{e,n}$

$$\therefore\ R = r_{c,n} - r_{e,n} = \sigma_n v_{th} N_t (n - n f_t - n_t f_t)$$

$$= \sigma_n v_{th} N_t \left[n - n\left(\frac{\sigma_n n + \sigma_p p_t}{\sigma_n(n + n_t) + \sigma_p(p + p_t)} \right) - n_t \left(\frac{\sigma_n n + \sigma_p p_t}{\sigma_n(n + n_t) + \sigma_p(p + p_t)} \right) \right]$$

$$= \sigma_n v_{th} N_t \left[\frac{np\sigma_p + \cancel{n p_t \sigma_p} + \cancel{n^2 \sigma_n} + \cancel{n n_t \sigma_n} - \cancel{n^2 \sigma_n} - \cancel{n p_t \sigma_p} - \cancel{n n_t \sigma_n} - n_t p_t \sigma_p}{\sigma_n(n + n_t) + \sigma_p(p + p_t)} \right]$$

$$= \sigma_n v_{th} N_t \left[\frac{np\sigma_p - n_t p_t \sigma_p}{\sigma_n(n + n_t) + \sigma_p(p + p_t)} \right] \quad ; \quad n_t p_t = n_i^2$$

$$\therefore\ \boxed{R = \sigma_n \sigma_p v_{th} N_t \left[\frac{np - n_i^2}{\sigma_n(n + n_t) + \sigma_p(p + p_t)} \right]}$$

2. a) $n >> p$, $\underbrace{p = p_0 + \Delta p \approx \Delta p}_{\text{as } n\text{-type}}$, $\underbrace{n = n_0 + \Delta n \approx n_0,\ \Delta p << n_0,\ n_i^2 << np}_{\text{as under LOW injection}}$

Also, as stated in the question, $n >> n_t, p_t$ (as defect levels near midgap).

Hence $R = \sigma_n \sigma_p v_{th} N_t \left[\dfrac{n_0 \Delta p - \overbrace{n_i^2}^{\text{negligible}}}{\sigma_n (n_0 + \underbrace{n_t}_{\text{negligible}}) + \sigma_p (\underbrace{\Delta p + p_t}_{\text{negligible}})} \right] \approx \sigma_n \sigma_p v_{th} N_t \left[\dfrac{n_0 \Delta p}{\sigma_n n_0} \right] = \sigma_p v_{th} N_t \Delta p$

$\therefore$ $\boxed{R \approx \dfrac{\Delta p}{\tau_p} \quad \text{where } \tau_p = \dfrac{1}{\sigma_p v_{th} N_t}}$

So the recombination rate is independent of the majority carrier concentration (n_0), and the lifetime τ_p only depends on the hole capture cross-section (σ_p), with no electron parameters. This is because there is an abundance of electrons in an n-type semiconductor, so as soon as a hole is captured by a defect level, an electron will immediately be captured to complete the recombination process. Thus, the rate limiting step in the recombination process is the capture of a MINORITY carrier.

b) Very similar to above, with the same logic: $\boxed{R \approx \dfrac{\Delta n}{\tau_n}, \quad \tau_n = \dfrac{1}{\sigma_n v_{th} N_t}}$

c) $$R = \frac{np - n_i^2}{\frac{1}{\sigma_p v_{th} N_t}(n + n_t) + \frac{1}{\sigma_n v_{th} N_t}(p + p_t)} = \boxed{\frac{np - n_i^2}{\tau_p (n + n_t) + \tau_n (p + p_t)}}$$

3. a) Again, n-type under low injection, $\therefore$ $n >> p$, $p \approx \Delta p$, $n \approx n_0$, $\Delta p << n_0$, $n_i^2 << np$

HOWEVER, n_t and p_t NO LONGER NEGLIGIBLE. Also, $\tau_p \approx \tau_n = \tau$

$$\therefore \quad R \approx \frac{n_0 \Delta p - \overbrace{n_i^2}^{\text{negligible}}}{\tau(n_0 + \underbrace{\Delta p}_{\text{negligible}} + n_t + p_t)}$$

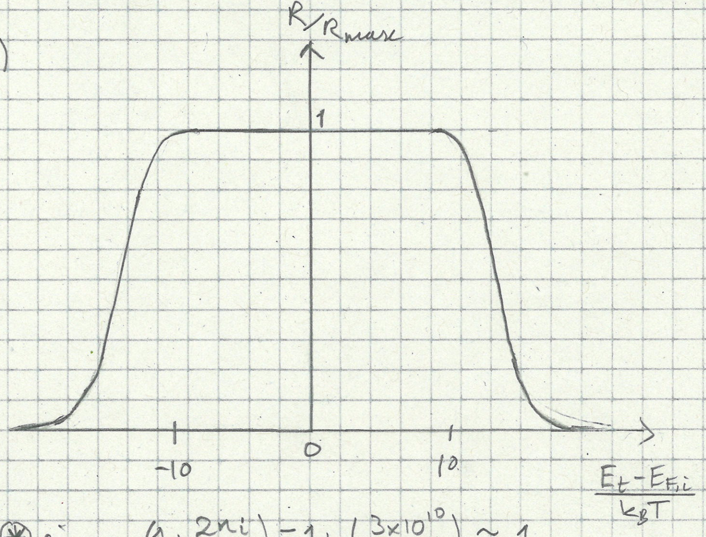
$$n_t + p_t = n_i e^{(E_t - E_{F,i})/k_B T} + n_i e^{-(E_t - E_{F,i})/k_B T} = 2 n_i \cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right)$$

$$\therefore \quad R \approx \frac{n_0 \Delta p}{\tau\left(n_0 + 2 n_i \cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right)\right)} = \frac{\Delta p}{\tau\left(1 + \left(\frac{2 n_i}{n_0}\right) \cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right)\right)}$$

$$\boxed{R \approx \frac{\Delta p}{\tau_r}, \quad \tau_r = \tau\left(1 + \left(\frac{2 n_i}{n_0}\right) \cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right)\right)}$$

So, we have an expression comparable to that found in 3(a), but now the lifetime is dependent on E_t and n_0, since we have not made the assumption that $E_t \approx E_{F,i}$.

b)



R will be a maximum when τ_r is at a minimum, i.e when $\cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right) = 1$, $\tau_r \approx \tau$ ⊛

Hence $R_{max} = \frac{\Delta p}{\tau}$ and:

$$\frac{R}{R_{max}} = \left(1 + \left(\frac{2n_i}{n_0}\right)\cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right)\right)^{-1}$$

The recombination rate has a BROAD maximum, within which varying E_t has no effect. Note that for higher majority carrier concentrations (n_0), this maximum becomes even broader.

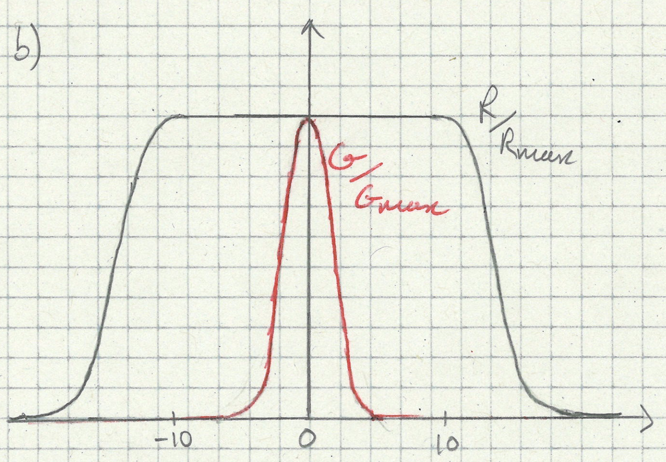
⊛ since $\left(1 + \frac{2n_i}{n_0}\right) = 1 + \left(\frac{3\times10^{10}}{1\times10^{15}}\right) \simeq 1$

4. a) Now, $n << n_i$, $p << n_i$ (and G_L is now a uniform EXTRACTION rate):

$$R = \frac{\cancel{np} - n_i^2}{\tau\left(\cancel{n} + \cancel{p} + 2n_i\cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right)\right)} \approx \frac{-n_i^{\cancel{2}}}{2\tau \cancel{n_i}\cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right)}$$

$$\therefore \boxed{G = -R = \frac{n_i}{\tau_g} \quad \text{where } \tau_g = 2\tau\cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right)}$$

b)



Similar to before, G will be a maximum when τ_g is a minimum, i.e $\tau_g = 2\tau$. Hence $G_{max} = \frac{n_i}{2\tau}$ and:

$$\frac{G}{G_{max}} = \left(\cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right)\right)^{-1}$$

We can see that G is much more sensitive to the variation of E_t, with a maximum at $E_t = E_{F,i}$, i.e. at the midgap. This is because the energy level plays a greater role in equalizing the rate of transfer between the defect level and the conduction/valence bands when carrier densities are small.

c) The above sketch shows us that the recombination rate can be much greater than the generation rate if the defect level energy is away from the midgap. This means device properties can be tuned INDEPENDENTLY by controlling the energy of the defect level. For instance, it is preferable to have high R and low G for a diode, as this will result in faster turn off and lower reverse bias current leakage (respectively).
Crucially, for a diode we DON'T want midgap defect levels as this greatly increases the generation rate in the depletion region (see question 6.)

5. • NO BAND TO BAND RECOMBINATION OR GENERATION

This assumption is reasonable for an indirect semiconductor such as silicon. For these semiconductors, a direct recombination process is very unlikely, because the electrons at the bottom of the conduction band have a non-zero momentum with respect to the holes at the top of the valence band. As such, a direct transition is not possible without also involving a lattice phonon (to provide the momentum). The defect levels act as a "stepping stone" in the bandgap, and allow for much faster recombination.

• NO AUGER RECOMBINATION

This is also a reasonable assumption, as Auger recombination is a three-body process and as such can only dominate at high carrier densities: & for all our problems we imagined LOW injection and MODERATE doping, so we shouldn't have high enough carrier densities.

• MODERATELY DOPED

We assumed this when we used the Boltzmann approximation to derive our expressions for the emission probabilities (very high doping can lead to degeneracy and the invalidity of the Boltzmann approximation).

• v_{th} THE SAME FOR ELECTRONS AND HOLES

Since $v_{th} \propto \frac{1}{\sqrt{m^*}}$, this amounts to assuming the effective masses of electrons and holes in our semiconductor are the same.

• $\tau_n \approx \tau_p = \tau$

Since $\tau \propto \frac{1}{\sigma}$, this amounts to assuming $\sigma_n = \sigma_p = \sigma_0$.

• DEFECT LEVELS DONT INTERACT

This should be true as long as the concentration, N_t, remains much lower than the atomic density of the semiconductor (i.e $N_t << 10^{22}\ cm^{-3}$).

• LOW INJECTION OF CARRIERS

• SYMMETRIC INJECTION

i.e electron-hole pair injection (e.g illumination), not just electron / hole injection.

Any more you can think of?

6. a) $\tau_g = 2\tau \cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right) = \frac{2}{v_{th}\sigma_0 N_t}\cosh\left(\frac{E_t - E_{F,i}}{k_B T}\right)$ ⑥

$N_t = 10^{14}\,cm^{-3}$: $\therefore \tau_g = \frac{2}{(2.63\times10^{-8})(10^{14})}\cosh\left(\frac{(0.02)(1.602\times10^{-19})}{(1.38\times10^{-23})(300)}\right) = 1.0\times10^{-6} \approx \underline{\underline{1\ \mu s}}$

$N_t = 10^{18}\,cm^{-3}$: $\tau_g = \frac{(1\times10^{-6})(10^{14})}{(10^{18})} = 1.0\times10^{-10} \approx \underline{\underline{0.1\ ns}}$

(b) Silicon is <u>very</u> sensitive to the presence of gold, since it generates an energy level so close to the midgap. A sharp reduction in generation lifetime as seen in 7. (a) would cause a large increase in the reverse bias leakage of a p-n junction diode since carriers would be generated in the depleted region at a much faster rate.
This could lead to unacceptable leakage in many silicon devices, and is the reason why gold is BANNED in any serious cleanrooms dealing with silicon (even though for other materials it is often used to make an ohmic contact).

SUPPLEMENTARY INFORMATION:

You should also be able to plot and understand τ_g and τ_r vs. $\frac{E_t - E_{F,i}}{k_B T}$: